Effects of Heat Transfer of Viscous Incompressible Dusty Fluid

Dr. Ravita Bhardwaj
Asst. Professor, Mathematics,
S.D. College, Palwal, Haryana

Abstract:- If the lines of motion are parallel to a fixed plane and if the velocity at corresponding points of all planes has the same magnitude and direction the motion is two dimensional and w=0. The present problem is concerned with heat and mass transfer in free convective flow of a visco elastic dusty gas through a porous medium induced by the motion of a semi-infinite flat plate moving with velocity decreasing exponentially with time heat source parameter. It is clear that the existence of a stream function is a consequence of stream lines and equation of continuity for incompressible fluid. The expression for velocity distribution of dusty fluid, dust particle, temperature and concentration distribution are obtained. Source is a point at which liquid is continuously created and sink is a point at which liquid is continuously annihilated.

Key Words: - Heat and Mass transfer, MHD, free convective porosity.

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Introduction: - The problem of single rectilinear vertex in an unlimited mass of liquid remains stationary and fluid mechanical involving gas particle mixture arise in many processes of practical importance. The study of magnet hydronamic flow for an electrically conducting fluid past a heated surface has attracted the interest of many researchers in view of its important applications in many engineering problem such as plasma studies, cooling of nuclear reactor, the boundary layer control in aerodynamics. The presence of such impurities is studies in the literature by considering it as foreign mass. Soundalgekar (1977) first presented an exact solution to the flow of a viscous to fluid past an impulsively started vertical plate. By using a multiscale analysis, the MHD equations can be reduced to a set of four closed scalar equations. This allows e.g. for more efficient numerical calculations.

Based on the MHD equations, Glatzmaier and Paul Roberts have made a supercomputer model of the earth's interior. The presence of such impurities is studied in the literature by considering it as a foreign mass. The study of convection with heat and mass transfer is very useful in fields such as chemistry =, agriculture, oceanography. A few representative fields of interest in which combined heat and mass transfer play an important role are the design of chemical processing equipment, formation and dispersion of fog, distribution of temperature and moisture over agriculture fields, damage of crops due to freezing and pollution of the environment. This technique is used in the cooling process of plastic sheets, polymer fiber, and glass materials and in drying process of paper.

In this present work, study of heat transfer of viscous, incompressible dusty fluid is studied. It starts vertical plate with heat source sink and with a constant velocity while the lower plate is kept stationary.

II Related Works

A number of scientists have carried out investigation in areas related to this study. Merkin & Mahmood (1990) investigated on free convection boundary layer on a vertical plate with prescribed surface heat flux while a study on Hydro magnetic free convection currents effects on boundary layer thickness was carried out by Marigi et al. (2010), Ram et. Al (1995) solved magnetic hydrodynamics stokes problem of convection flow for a vertical infinite plate in dissipative rotating fluid with hall current. They analyzed the effects of various parameters on the concentration velocity and temperature profiles. A steady MHD flow of an electrically conducting fluid between parallel infinite plates was done by Chandra (2005), while Kwanza et al; (2003) presented their work on MHD stokes free convection past an infinite vertical porous. Plate subjected to a constant heat flux with radiation absorption. They discussed their tabulated results on concentration, velocity and temperature distributions both theoretically and graphically. They discussed the effects of modified gras of number, heat source parameter, Schmidt number, time, and hall current, angle of inclination and Ec kert number on a convectively heated plate restricted to a laminar boundary layer. They found that an increase in mass diffusion parameter causes a decrease in concentration profiles while an increase in suction velocity leads to an increase in concentration profiles.

FORMULATION OF PROBLEM:-

A study of a viscous, incompressible and electrically conducting fluid flowing between two horizontal plates separated at distance h is considered. The unsteady dusty flow of a viscous incompressible, slightly conducting fluid past an impulsively started infinite vertical plates is considered. The X-axis is taken along the plate in the vertically upward direction and the Y-axis is taken normal to the plate at time t<=0, the plate, the fluid and the dust particle are at the same temperature & concentration in a stationary condition. At time t>0, the plate is given an impulsive motion in the vertical direction with constant velocity w, at the plate constant heat & mass flux as imposed at u be the fluid velocity and v be the velocity of dust particle in direction of x-axis. Then the unsteady flow past an infinite vertical plate is governed by the following equations:-

$$\frac{\partial u'}{\partial t'} = g\beta(T' - T'_\infty) + g\beta^*(C' - C'_\infty) + \nu\frac{\partial^2 u'}{\partial y^2} - \nu\left(\frac{u'}{k'}\right) \quad (1)$$

$$\rho c_p \frac{\partial T'}{\partial t'} = \kappa \frac{\partial^2 T'}{\partial y^2} \quad (2)$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y^2}$$

Where the symbols have their usual meanings.

Assumptions: - The governing equations are written based on the following assumptions:-

- The dust particle are solid, spherical, non-conducting, equal in size, and uniformly distributed in the flow region. This means that the dust particle gain heat energy from the fluid by conduction through their spherical surface.
- The number density of dust particle is constant and the temperature between the particles is uniform throughout the motion. It is an incompressible fluid, therefore the density is constant and also to prevent energy loss between the particles.

- The buoyancy force, induced magnetic field and Hall effects, have been neglected. This means that flow regions has uniform temperature, uniform applied magnetic field and a Cartesian co-ordinates.
- The interactions between the particles, chemical reaction and radiation between the particles and liquid have not been considered. This is necessary in order to avoid multiple equations.
- The magnetic Rey holds number is taken to be very small and so that induced magnetic field is negligible. This means that a uniform magnetic field Bo is applied in the positive Y direction and is the only magnetic field in the problem.
- The volume occupied by the particle per unit volume of mixture (i.e. volume fraction of dust particle and mass concentration have been taken into consideration)
- The dust concentration is so small so that it is not disturbing the continuity and hydro magnetic effects. This means that the continuous equation is satisfied.

Governing Equations: - The fluid flow is governed by the energy equations under the above assumptions:-

$$(1-\phi)\frac{\partial u}{\partial t}=(1-\phi)\left[-\frac{1}{\rho}\frac{\partial p}{\partial x}+v\frac{\partial^2 u}{\partial y^2}+g\beta^{+}(T-T_0)\right]+\frac{KN_0}{\rho}(v-u)+\frac{KN\sigma\mu_c^2 H_0^2}{\rho}u-\frac{\mu}{K_1}u$$

$$N_0 m\frac{\partial v}{dt}=\phi\left[\frac{\partial p}{\partial x}+\mu\frac{\partial^2 u}{\partial y^2}+\rho g\beta^{+}(T-T_0)\right]+KN_0(u-v)$$

$$\frac{\partial T}{\partial t}=\frac{\kappa}{\rho C_p}\frac{\partial^2 T}{\partial y^2}$$

The boundary conditions to the problem are:-

$$t\le 0;\quad u(y,t)=v(y,t)=0,\quad T(y,t)=0\quad for\quad 0\le y\le 1$$

$$t>0;\quad u(y,t)=v(y,t)=0,\quad T(y,t)=0\quad at\quad y=0$$

$$u(y,t)=v(y,t)=1+\varepsilon e^{\text{int}}\quad T(y,t)=1+\varepsilon e^{\text{int}}\quad at\quad y=1$$

Where u (y, t) is the velocity of the fluid and v (y, t) is velocity of dust particles, m is the mass of each dust particle. NO is the number density of dust particle, T is the temperature. T0 is the initial temperature. Tw is the raised temperature, βt is the volumetric coefficient of thermal expansion. Cp is the specific heat at constant pressure, Ø is the volume fraction of dust particle (i.e. the volume occupied by the particles per unit volume of the mixture, K is the Stokes resistance coefficient C= 6πur for special particles of radius r), Ho is the magnetic field induction, Uc is magnetic permeability, σ is the electrical conductivity of the liquid, K is the thermal conductivity and K1 is the porous parameter. The first term consists of pressure gradient while the second is the viscous flow and the third buoyancy force terms respectively. The last three terms represent the force term due to the relative motion between fluid and dust particles, magnetic and porous terms respectively while the left hand side represent stream wise velocity unsteady term. From equation 2, the left hand side signifies unsteady normal velocity expressed in terms of pressure and viscous dissipation terms.

The problem is simplified by writing the equations in the non-dimensional form. The characteristic length is taken to be h and characteristic velocity is V. We introduce the following non dimensional variables:-

$$x^*=\frac{x}{h}, y^*=\frac{y}{h}, p^*=\frac{h^2 p}{\rho v^2}, t^*=\frac{vt}{h^2}, u^*=\frac{uh}{v}, T^*=\frac{T-T_0}{T_w-T_0}$$

$G_r=\frac{g\beta^+(T_w-T_0)h^3}{v^2}$ (Grashofnumber), $\varepsilon_1=\frac{f}{\sigma_1(1-\phi)}$,

$\sigma_1=\frac{mv}{Kh^2}$, $\varepsilon_2=\frac{1}{1-\phi}$, $M=\mu_c^2 h^2 H_0^2\frac{\sigma}{\mu}$ (Magnetic parameter), $f=\frac{mN_0}{\rho}$ (mass concentration of dust particles), $\varepsilon_3=\frac{\mu h^2}{K_1(1-\phi)}$ (Porous Parameter), $\beta=\frac{f}{\sigma_1}$ (concentration resistance ratio)

The non-dimensional boundary conditions are:

$$t\le 0;\quad u(y,t)=v(y,t)=0,\quad T(y,t)=0 \quad for\ 0\le y\le 1.$$

$$t>0;\quad u(y,t)=v(y,t)=0 \quad T(y,t)=0 \quad at \quad y=0.$$

$$u(y,t)=v(y,t)=1+\varepsilon e^{int},\quad T(y,t)=1+\varepsilon e^{int},\quad at \quad y=1.$$

Method of Solution:-

At t>0, the temperature and concentration level changes according to the following laws:-

$$T'=T'_\infty+(T'_w-T'_\infty).e^{-at'}$$

$$C'=C'_\infty+(C'_w-C'_\infty).e^{-at'}$$

Where, 'a' is the decay factor.

The initial and boundary condition relevant to the problem are:

$$\left.\begin{array}{lllll} t'\le 0: & u'=0=v', & T'=T'_\infty, & C'=C'_\infty, & for\ all \quad y'\le 0 \\ t'>0: & u'=we^{-at}=v', & T'=T'_\infty+(T'_w-T'_\infty).e^{-at'}, & C'=C'_\infty+(C'_w-C'_\infty).e^{-at'} & for \quad y'=0 \\ & u'=0=v', & T'\to T'_\infty, & C'\to C'_\infty, & as \quad y'\to\infty \end{array}\right\}$$

We introduce the following non dimensional quantities:

$$u=\frac{u'}{w},\quad v=\frac{v'}{w},\quad y=\frac{wy'}{\upsilon},\quad t=\frac{w^2t'}{\upsilon},\quad l=\frac{mN_0}{\rho},\quad \lambda=\frac{mw^2}{K_0\upsilon}$$

$$K=\frac{w^2K'}{\upsilon^2},\quad Sc=\frac{\upsilon}{D},\quad \Pr=\frac{\mu C_p}{k_T},\quad \theta=\frac{T'-T'_\infty}{T'_w-T'_\infty}$$

$$\phi = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \qquad A = \frac{D_1 (T'_w - T'_\infty)}{\upsilon (C'_w - C'_\infty)}, \qquad Gr = \frac{g\beta\upsilon(T'_w - T'_\infty)}{w^3}$$

$$Gm = \frac{g\beta^* \upsilon (C'_w - C'_\infty)}{w^3}, \qquad M = \frac{\sigma B_0^2 \upsilon}{\rho w^2}, \qquad E_0 = \frac{w^2 E'_0}{\upsilon^2}$$

Introducing these non- dimensional quantities, the equations reduce to:

$$\frac{\partial u}{\partial t} = Gr\theta + Gm\phi + \left(1 + E_0 \frac{\partial}{\partial t}\right)\frac{\partial^2 u}{\partial y^2} + \frac{l}{\lambda}(v - u) - \left(M + \frac{1}{K}\right)u$$

$$\lambda \frac{\partial v}{\partial t} = (u - v)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2}$$

$$\frac{\partial \phi}{\partial t} = \frac{1}{Sc}\frac{\partial^2 \phi}{\partial y^2} + A\frac{\partial^2 \theta}{\partial y^2}$$

The boundary conditions are reduced to:

$$\left.\begin{array}{lll} t \le 0: & u = 0 = v, \quad \theta = 0, \quad \phi = 0 & for\ all \quad y \le 0 \\ t > 0: & u = e^{-at} = v, \quad \theta = e^{-at}, \quad \phi = e^{-at} & for\ all \quad y = 0 \\ & u = 0 = v, \quad \theta \to 0, \quad \phi \to 0 & as \quad y \to \infty \end{array}\right\}$$

Let us choose the general solution

$$G(y,t) = G_0(y).e^{-at}$$

By virtue of equation (13), the equations (8) to (11) transform respectively.

$$\frac{d^2 u_0}{dy^2} + n^2 u_0 = -\frac{1}{(1 - aE_0)}(Gr\theta_0 + Gm\phi_0)$$

$$v_0 = \left(\frac{1}{1 + \lambda a}\right)u_0$$

$$\frac{d^2 \theta_0}{dy^2} + a\Pr\theta_0 = 0$$

$$\frac{d^2 \phi_0}{dy^2} + aSc\phi_0 = -A\frac{d^2 \theta_0}{dy^2}$$

Where $$n = \frac{1}{\sqrt{1-aE_0}}\left[\frac{a\{(1+l)-a\lambda\}}{(1-\lambda a)} - \left(M + \frac{1}{K}\right)\right]^{\frac{1}{2}}$$

The boundary conditions (12) are transformed:

$$\left.\begin{array}{llll} u_0 = 1 = v_0, & \theta_0 = 1, & \phi_0 = 1 & for\ all \quad y = 0 \\ u_0 = 0 = v_0, & \theta_0 \to 0, & \phi_0 \to 0 & at \quad y \to \infty \end{array}\right\}$$

By the condition , the solutions are obtained and the real parts of velocity profiles, temperature and concentration profiles are given as:

$$u = \Big[\ \cos ny + \frac{Gr(1-\Pr) + GmA\Pr}{(1-aE_0)(n^2 - a\Pr)(1-\Pr)}\left\{\cos ny - \cos\sqrt{a\Pr}\,y\right\} + \frac{Gm}{(1-aE_0)(n^2 - aSc)}\left\{\cos ny - \cos\sqrt{aSc}\,y\right\}\ \Big]\, e^{-at}$$

$$v = \frac{1}{(1+a\lambda)}\Big[\ \cos ny + \frac{Gr(1-\Pr) + GmA\Pr}{(1-aE_0)(n^2 - a\Pr)(1-\Pr)}\left\{\cos ny - \cos\sqrt{a\Pr}\,y\right\} + \frac{Gm}{(1-aE_0)(n^2 - aSc)}\left\{\cos ny - \cos\sqrt{aSc}\,y\right\}\ \Big]$$

$$\theta = \cos\sqrt{a\Pr}\,y.e^{-at}$$

$$\phi = \left[\left\{1 - \frac{A\Pr}{(1-\Pr)}\right\}\cos\sqrt{aSc}\,y. + \frac{A\Pr}{(1-\Pr)}\cos\sqrt{a\Pr}\,y\right].e^{-at}$$

Result and Discussions:-

The fluid velocity profile of dusty fluid and dust particle are tabulated in graphs for Grashoff number, Gr=2, Gm=3, Schmidt number Sc=0.4, Prandtl number Pr=0.71, l=0.5, decay factor a =0.3. Permeability parameter K=4, magnetic field parameter M =0.1 and different values of time t.

It is observed that velocity decreases continuously with increasing y and increases with increases time t. It is found that velocity of dust particle decreases with velocity of dusty fluid. The exponential dependence of viscosity on temperature results in decomposing the viscous force term into two terms. The variation of these resulting terms with the viscosity parameter and their relative magnitude have an important effect on the flow and temperature fields in the absence or presence of the applied uniform magnetic field. In the following discussion selected parameters are given, the following fixed values:-

R=0.5, G=0.8, ∞=5, Pr=1, Ec=0.2 and Lo=0.7

The velocity profile of dusty fluid of boundary layer flow are tabulated in table 4 and plotted graphs in figure 4 for permeability parameter r=0.3, Magnetic field parameter M=0.1, time t=1. Prandtl number Pr=0.71, Schmidt number Sc=0.4 and decay factor a=0.3, different values of Grashoff numbers Gr and Gm. It is observed that the velocity of dusty fluid decreases with increasing Gr and Gm.792

The temperature profile of a boundary layer flow is plotted tabulated in Table-5 and plotted in Graph-5 for Gr=5, Gm=2, M=0.1, K=3, Schmidt number Sc=0.6, L=0.5, r=0.5, a=0.3, t=1 and different values of Prandtl number Pr and decay factor a. It is observed that the temperature decreases with increasing Prandtl number Pr and decay factor a.

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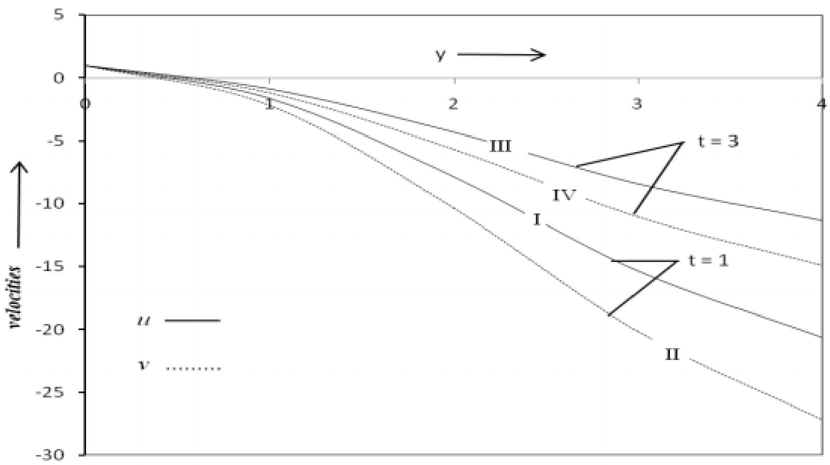
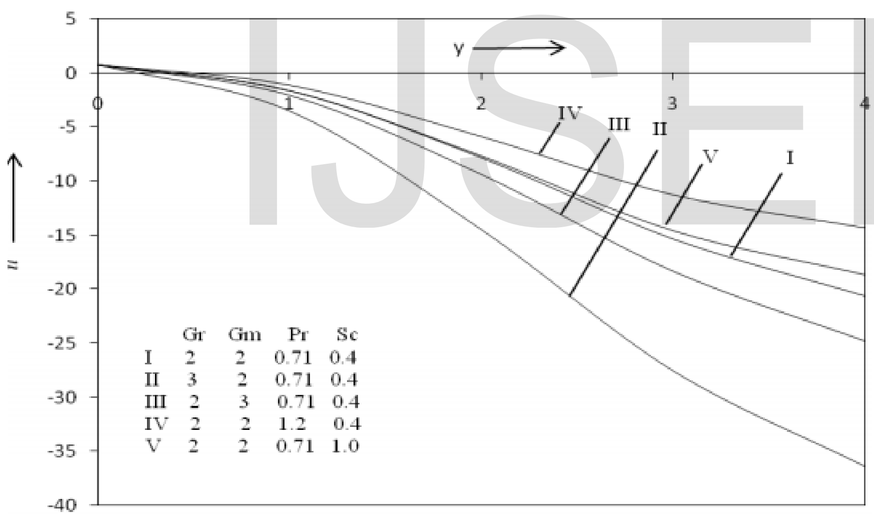
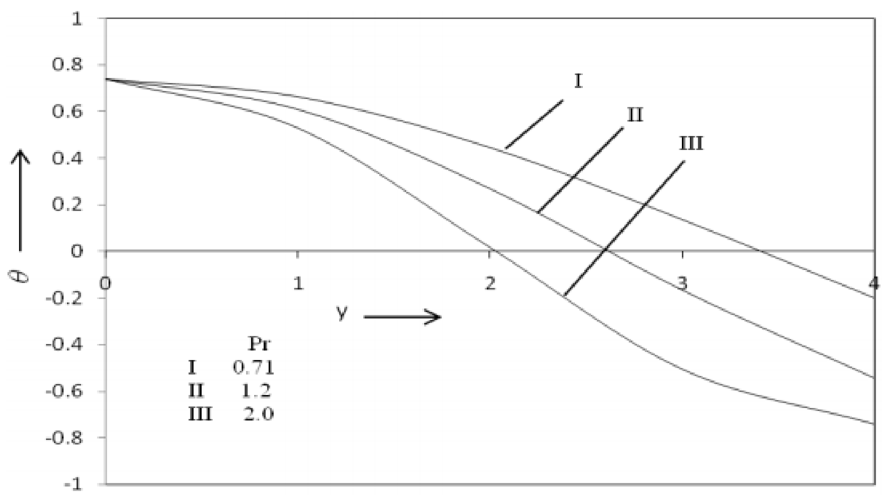


The velocity profiles for different value of time t.



The velocity profile for different value of Gr, Gm , Pr and Sc.



. The temprature profile for the different value of Pr.

Conclusion:-

Study of flow of a dusty viscous incompressible, electricity conducting and fluid velocity profile of dusty fluid and dust particle increases with increasing time and velocity of dust particle decreases with velocity of dusty fluid. The velocity of dusty fluid decreases with increasing M and increases with increasing K. The velocity profile of dusty fluid decreases with increasing Gr and Gm. The concentration profile of dusty fluid decreases with increasing Sc and decay factor a.

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